1) 
\[ \int_0^1 \left( 2x + 4 + \frac{2}{x^2} + x \right) \, dx = 6 \left( \frac{1}{2} + 4.2 + 4.3 + 0.5 \right) \]

\[ = 6 \left( 13.1 \right) \text{ without units} = 78.6 \]

(b) Since \( R(t) \) is differentiable and \( R'(t) = 9.6 \), by Rolle's Theorem, there exist some \( c \in [0, 1] \) such that \( R'(c) = 0 \). The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function \( R \) of time \( t \). The table above shows the rate as measured every 3 hours for a 24-hour period.

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate \( \int_0^3 R(t) \, dt \). Using correct units, explain the meaning of your answer in terms of water flow.

(b) Is there some time \( t \), \( 0 < t < 24 \), such that \( R'(t) = 0 \)? Justify your answer.

(c) The rate of water flow \( R(t) \) can be approximated by \( Q(t) = \frac{1}{9.6} \left( 130 + 23t - t^2 \right) \).

Use \( Q(t) \) to approximate the average rate of water flow during the 24-hour time period.

Indicate units of measure.

Any rate of water flow = avg value of a function here

2)

(b) \( w(a) = \frac{w(a) - w(b)}{a - b} = \frac{21 - 24}{15} = \frac{-3}{15} = -0.2 \text{ C/°}

The temperature, in degrees Celsius, of the water in a pond is a differentiable function \( W \) of time \( t \). The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for \( W'(12) \). Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval \( 0 \leq t \leq 15 \) days by using a trapezoidal approximation with subintervals of length \( \Delta t = 3 \) days.

(c) A student proposes the function \( P \), given by \( P(t) = 20 + 10e^{-1/15} \), as a model for the temperature of the water in the pond at time \( t \), where \( t \) is measured in days and \( P(t) \) is measured in degrees Celsius. Find \( P'(12) \). Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function \( P \) defined in part (c) to find the average value, in degrees Celsius, of \( P(t) \) over the time interval \( 0 \leq t \leq 15 \) days.
The graph of a differentiable function $f$ on the closed interval $[-3, 15]$ is shown in the figure above. The graph of $f$ has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_{-3}^{x} f(t) \, dt$ for $-3 \leq x \leq 15$.

(a) Find $g(0)$, $g'(0)$, and $g''(0)$.

(b) On what intervals is $g$ decreasing? Justify your answer.

(c) On what intervals is the graph of $g$ concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t) \, dt$ using six subintervals of length $\Delta t = 3$.

\[
\int_{-3}^{15} f(t) \, dt \approx 2 \left[ \frac{3}{2} (119) \right] = 118
\]

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time $t$ minutes, where $v$ is a differentiable function of $t$. Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_{0}^{40} v(t) \, dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_{0}^{40} v(t) \, dt$ in terms of the plane’s flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 15$? Justify your answer.

(c) The function $f$, defined by $f(t) = 6 + \cos \left( \frac{t}{10} \right) + 3 \sin \left( \frac{7t}{20} \right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.

(d) According to the model $f$, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

\[
\frac{1}{40} \int_{0}^{40} v(t) \, dt = 5.916
\]
A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature \( T(x) \), in degrees Celsius (°C), of the wire \( x \) cm from the heated end. The function \( T \) is decreasing and twice differentiable.

(a) Estimate \( T'(7) \). Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of \( T(x) \) for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find \( \int_{0}^{8} T(x) \, dx \), and indicate units of measure. Explain the meaning of \( \int_{0}^{8} T'(x) \, dx \) in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that \( T''(x) > 0 \) for every \( x \) in the interval \( 0 < x < 8 \) ? Explain your answer.